OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4728

Mechanics 1

Friday **21 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages and 3 blank pages.

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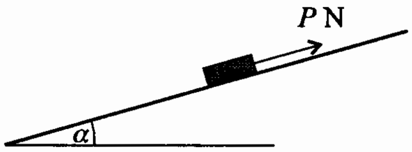
1



A box of weight 100 N rests in equilibrium on a plane inclined at an angle α to the horizontal. It is given that $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$. A force of magnitude P N acts on the box parallel to the plane in the upwards direction (see diagram). The coefficient of friction between the box and the plane is 0.25.

(i) Find the magnitude of the normal force acting on the box. [2]

(ii) Given that the equilibrium is limiting, show that the magnitude of the frictional force acting on the box is 24 N. [1]

(iii) Find the value of P for which the box is on the point of slipping

(a) down the plane,

(b) up the plane.

[3]

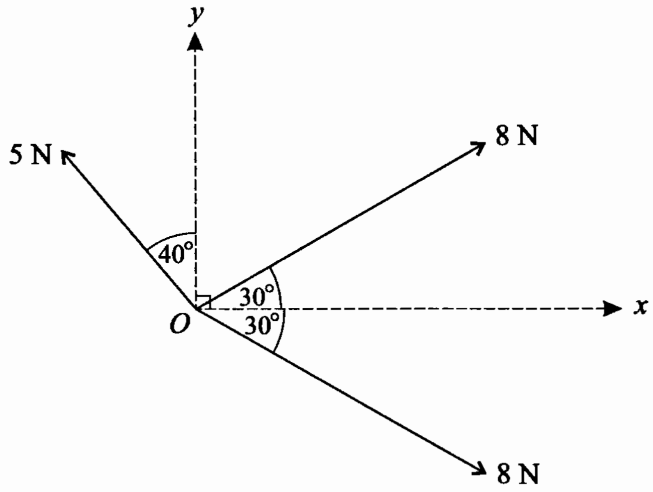
2

Three small uniform spheres A, B and C have masses 0.4 kg, 1.2 kg and m kg respectively. The spheres move in the same straight line on a smooth horizontal table, with B between A and C. Sphere A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$, B is moving towards A with speed $2\,\mathrm{m\,s^{-1}}$ and C is moving towards B with speed $4\,\mathrm{m\,s^{-1}}$ (see diagram). Spheres A and B collide. After this collision B moves with speed $1\,\mathrm{m\,s^{-1}}$ towards C.

(i) Find the speed with which A moves after the collision and state the direction of motion of A. [5]

(ii) Spheres B and C now collide and move away from each other with speeds $0.5\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively. Find the value of m. [3]

3



Three coplanar forces of magnitudes 5 N, 8 N and 8 N act at the origin O of rectangular coordinate axes. The directions of the forces are as shown in the diagram.

(i) Find the component of the resultant of the three forces in

(a) the x-direction,

(b) the y-direction. [5]

(ii) Find the magnitude and direction of the resultant. [4]

4 A particle moves in a straight line. Its velocity t s after leaving a fixed point on the line is $v\,\mathrm{m\,s^{-1}}$, where $v = t + 0.1t^2$. Find

(i) an expression for the acceleration of the particle at time t, [2]

(ii) the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $2.8\,\mathrm{m\,s^{-2}}$. [7]

5 Two particles A and B are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of A and B are $7\,\mathrm{m\,s^{-1}}$ and $10.5\,\mathrm{m\,s^{-1}}$ respectively.

(i) Write down expressions for the heights above the ground of A and B at time t seconds after projection. [1]

(ii) Hence find a simplified expression for the difference in the heights of A and B at time t seconds after projection. [1]

(iii) Find the difference in the heights of A and B when A is at its maximum height. [3]

At the instant when B is 3.5 m above A, find

(iv) whether A is moving upwards or downwards, [3]

(v) the height of A above the ground. [2]

6

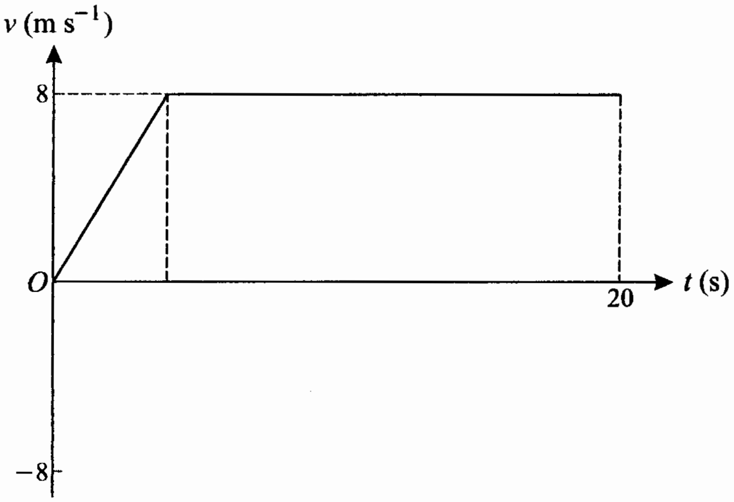


Fig. 1

A cyclist P travels along a straight road starting from rest at A and accelerating at 2 m s^{-2} up to a speed of 8 m s^{-1}. He continues at a constant speed of 8 m s^{-1}, passing through the point B 20 s after leaving A. Fig. 1 shows the (t, v) graph of P's journey for $0 \leqslant t \leqslant 20$. Find

(i) the time for which P is accelerating, [2]

(ii) the distance AB. [3]

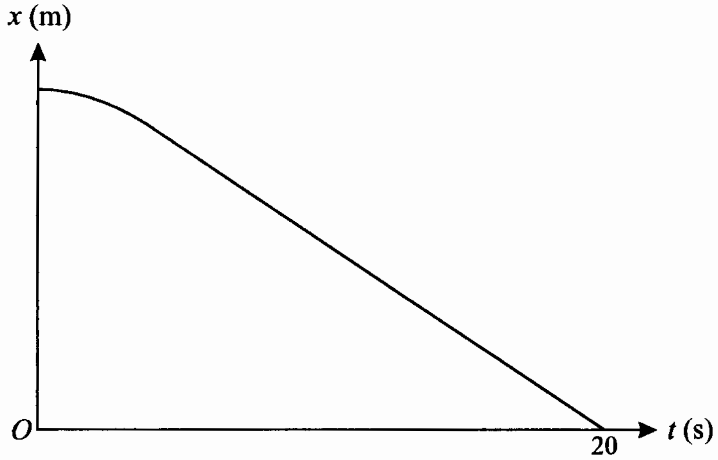


Fig. 2

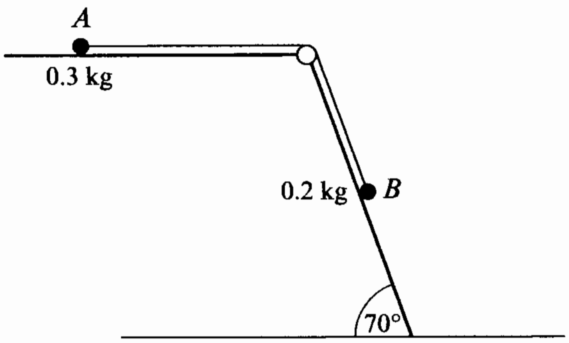
Another cyclist Q travels along the same straight road in the opposite direction. She starts at rest from B at the same instant that P leaves A. Cyclist Q accelerates at 2 m s^{-2} up to a speed of 8 m s^{-1} and continues at a constant speed of 8 m s^{-1}, passing through the point A 20 s after leaving B. Fig. 2 shows the (t, x) graph of Q's journey for $0 \leqslant t \leqslant 20$, where x is the displacement of Q from A towards B.

(iii) Sketch a copy of Fig. 1 and add to your copy a sketch of the (t, v) graph of Q's journey for $0 \leqslant t \leqslant 20$. [2]

(iv) Sketch a copy of Fig. 2 and add to your copy a sketch of the (t, x) graph of P's journey for $0 \leqslant t \leqslant 20$. [3]

(v) Find the value t at the instant that P and Q pass each other. [3]

7



The upper edge of a smooth plane inclined at 70° to the horizontal is joined to an edge of a rough horizontal table. Particles A and B, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth pulley which is fixed at the top of the smooth inclined plane. Particle A is held in contact with the rough horizontal table and particle B is in contact with the smooth inclined plane with the string taut (see diagram). The coefficient of friction between A and the horizontal table is 0.4. Particle A is released from rest and the system starts to move.

(i) Find the acceleration of A and the tension in the string. [8]

The string breaks when the speed of the particles is $1.5\ \text{m s}^{-1}$.

(ii) Assuming A does not reach the pulley, find the distance travelled by A after the string breaks. [3]

(iii) Assuming B does not reach the ground before A stops, find the distance travelled by B from the time the string breaks to the time that A stops. [6]

1	**(i)**	$R = W\cos\alpha$ Magnitude is 96 N	M1 A1 2	For resolving forces perpendicular to the plane
	(ii)	Magnitude is 24 N	B1 1	AG From correct work.
	(iii)	$P = 100\times0.28 - 24$ $P = 100\times0.28 + 24$ (a) $P = 4$ (b) $P = 52$	M1 A1 A1 3	For resolving 3 forces parallel to the plane (either case)

2	**(i)**	Momentum of *A* and *B* before collision = $0.4\times6 - 1.2\times2$ Momentum of *A* and *B* after collision = $0.4v + 1.2 \times 1$ $0.4\times6 - 1.2\times2 = 0.4v + 1.2 \times 1$ ($v = -3$) Speed is 3 ms^{-1} Direction is away from *B*	 B1 B1 M1 A1 A1 ft 5	Alternatively: Momentum lost by $A = 0.4\times(6 - v)$ B1 Momentum gained by *B* $= 1.2\times(1 + 2)$ B1 For using the principle of conservation of momentum Positive answer only ft from *v*
	(ii)	$1.2\times1 - 4m = -1.2\times0.5 + 2m$ or $1.2\times1 + 1.2\times0.5 = 4m + 2m$ $m = 0.3$	 B1 B1 B1 3	For momentum equation :- with lhs correct with rhs correct
				SR If mgv used for momentum instead of mv, then (i) Speed is 3 ms^{-1} B1 Direction is away from *B* B1 ft (ii) m = 0.3 B1

3	**(i)(a)**	 $X = 2\times8\cos30^o - 5\sin40^o$ Component is 10.6 N	M1 A1 A1 ft	For resolving 3 forces parallel to the *x*-axis ft for 4.17 from sin/cos mix only
	(i)(b)	$Y = 5\cos40^o$ Component is 3.83 N	B1 B1 ft 5	 ft for 3.21 from sin/cos mix only
	(ii)	$R^2 = 10.64^2 + 3.83^2$ Magnitude is 11.3 N $\tan\theta = 3.83/10.64$ Direction is 19.8^o anticlockwise from +ve *x*-axis	M1 A1 ft M1 A1 ft 4	For using $R^2 = X^2 + Y^2$ For using $\tan\theta = Y/X$

4	**(i)**		M1	For using $a = \dot{v}(t)$
		Acceleration is $1 + 0.2t$	A1 2	
	(ii)		M1	For solving $a(t) = 2.8$ for t
		$t = 9$	A1	
			M1*	For integrating $v(t)$ to find $s(t)$
			A1	For $t^2 \div 2$ correct in $s(t)$
			A1	For $t^3 \div 30$ correct in $s(t)$
		$s(9) = 9^2 \div 2 + 9^3 \div 30 - (0 + 0)$	dep*M1	For correct use of limits or equivalent
		(= 40.5 + 24.3)		
		Distance is 64.8 m	A1 ft 7	ft their $a = \dot{v}(t)$ from (i)

5	**(i)**	Heights are $7t - ½ gt^2$ and $10.5t - ½ gt^2$	B1 1	
	(ii)	Expression is $3.5t$	B1 1	From correct (i)
	(iii)	$0 = 7 - 9.8t$	M1	For using $v = u - gt$ with $v = 0$
		$t = 5/7$ or 0.714	A1	
		Difference is 2.5 m	A1 ft 3	ft value of t
	(iv)	$t = 1$	B1 ft	For using ans(ii) = 3.5 correctly
		Greater than 5/7 (may be implied) or $7 - g\times 1$ is -ve	M1	For comparing this t with the time to greatest height or considering the sign of v_A for this t
		Direction is downwards	A1 3	
	(v)	$h_A = 7\times 1 - ½\ 9.8\times 1^2$	M1	For using $h = ut - ½ gt^2$ with relevant t
		Height is 2.1 m	A1 2	

6	**(i)**		M1		For using the idea that the gradient represents acceleration or for using $v = u + at$
		Accelerating for 4 s	A1	2	
	(ii)		M1		For using the idea that the distance is represented by the area of the trapezium or using suitable formulae for the two stages of the journey
		$AB = ½ (16 + 20)8$	A1ft		
		Distance is 144 m	A1	3	
	(iii)		B1		Graph is single valued and continuous and consists of two straight line segments with one segment from the origin and the other parallel to the t axis
			B1	2	Graph for Q is the reflection of the graph for P in the t axis
	(iv)		B1		Graph is single valued and continuous and consists of two parts, one of which is a straight line segment,with x increasing from 0 for the interval $0 < t < 20$
			B1		$x_P(20)$ appears to be equal to $x_Q(0)$
			B1	3	Graph for P appears to be the reflection in $x =$ ans(ii) $\div$ 2 of graph for Q
	(v)	t = 20 – (½ 144 $\div$ 8) or 16 + 8(t-4) = 128 – 8(t-4) or equivalent	M1		For complete method of finding the required time
		Value of t is 11	A2	3	
					SR Allow B1 for t = 11 without explanation

7	**(i)**		M1		For applying Newton's second law to either particle
		$T – F = 0.3a$	A1		
		$0.2g\sin70^o - T = 0.2a$	A1		
		$R = 0.3g$	B1		
		$F = 0.4(0.3g)$	M1		For using $F = \mu R$
		$0.2g\sin70^o - 0.4(0.3g) = 0.5a$	M1		For eliminating F and T or a
		Acceleration is 1.33 ms^{-2}	A1		
		Tension is 1.58 N	A1	8	
	(ii)	$a = -0.4g$	B1		May be scored in (iii)
		$0 = 1.5^2 – 2\times3.92s$	M1		For using $v^2 = u^2 + 2as$ with $v = 0$
		Distance is 0.287 m	A1	3	
	(iii)		M1		For using $v = u + at$ or equivalent with $v = 0$ for A
		$0 = 1.5 – 3.92t$	A1ft		ft value of a from (ii)
		$t = 0.383$ (may be implied)	A1		
		$a = g\sin70^o$	B1		For acceleration of B
		$s = 1.5(0.383) + ½9.8\sin70^o(0.383)^2$ (= 0.574 + 0.674)	M1		For using $s = ut + ½ at^2$ or equivalent with $u \neq 0$
		Distance is 1.25 m	A1	6	